Table 1 Initial guess values

Case no.	a, km	e	i, deg	ω, deg	Ω, deg	E deg
1	6983.0	.044	40.27	172.1	275.78	0.0
2	6980.0	.042	40.0	172.0	275.0	0.0
3	6975.0	.04	40.0	171.0	275.0	0.0
4	6950.0	.025	35.0	172.0	265.0	3.0
5	6900.0	.025	30.0	170.0	260.0	10.0
Nominal	6983.21	.0437	40.27	172.13	275.777	0.0

Table 2 Typical performance of various optimization methods for orbit determination

Method	Case 1	Case 2	Case 3	Case 4	Case 5
DSC	C (16)[a]	F[b]	F	F	F
Brent	C (10)	C (18)	C (20)	F	F
Powell	C (7)	C (14)	C (18)	F	F
PHS	C (20)	F	F	F	F
CG	C (2)	F	F	F	F
Stewart	C (6)	C (4)	C (4)	C (19)	C (16)
Fletcher	C (6)	C (4)	C (4)	C (12)	C (14)
DCG	C (2)	C (2)	C (2)	F	F
GLS	C (1)	C (1)	C (4)	C (12)	F
CDCM	C (2)	C (2)	C (2)	F	F

[a] C = convergence (time taken for convergence is given, in parentheses, in minutes on an IBM 360 computer). [b] F = failure to converge.

Table 3 Range of initial guesses for which convergence is obtained

Element	CDCM	GLS	Stewart/ Fletcher
a, km	(6983, 6983.4)	(6981, 6985)	(6720, 7350)
e	(.04, .0475)	(.015, .07)	(0, 0.45)
i, deg	(40, 40.5)	(25, 50)	(0, 180)
ω, deg	(171.9, 172,4)	(171.5, 172.8)	(0, 360)
Ω, deg	(275.6, 275.9)	(275, 277)	(0, 360)
E, deg	0 ± .25	0 ± .8	0 ± 180

Gaussian noise is added to these analytical values before subjecting them to processing. Details of these simulations are given in Ref. 11. Table 1 gives five sets of values of orbital elements which are used as initial guesses. A typical performance of various methods for single-pass data is shown in Table 2.

Table 2 shows that the performance of the gradient-independent methods (DSC, Brent, Powell, and PHS) is inferior to that of the gradient-dependent methods. Most of the methods converge in cases 1-3, where the initial values are comparatively nearer to the actual orbital elements. The classical differential correction method (CDCM) and the two methods based thereon (DCG, GLS) failed when the initial values are far away from the actual elements. However, when successful, these methods converge quickly. Of these three methods, the Greenstadt correction with linear search (GLS) performed best. The Stewart and Fletcher methods turn out to be the best among all in the sense that they converged even when the initial conditions are far off. Fletcher's dual variable metric method is the more preferable of the two, since it takes less time.

A study is made to find the convergence range of Fletcher's method. A single element is varied while the remaining five are taken with their actual values. Comparison with classical differential correction method is given in Table 3 for a typical analysis of three-pass data.

These studies show that, for the problem of orbit estimation, variable metric and related optimization methods present much better convergence characteristics than the differential correction methods. The extension in the convergence region by using nonlinear optimization methods enhances the success of obtaining a better estimate of state vector in a shorter tracking period.

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Stability of Spacecraft During Asymmetrical Deployment of Appendages

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Nomenclature

$a_1(t), a_2(t)$ = time-varying terms in the differential equations for h_1 and h_2
c = boom extension rate
h_1, h_2, h_3 = components of the angular momentum vector along principal axes

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h_0 = assumed constant value of h_3 during nominal deployment maneuver
I_1, I_2, I_3 = instantaneous values of principal moments of inertia
I_1^*, I_2^*, I_3^* = hub principal moments of inertia
$l(t)$ = instantaneous length of the boom from the center of the spacecraft hub
$\dot{l}(t)$ = extension rate of the boom
m = mass of the end mass
$p(t), q(t)$ = time varying coefficients in the equations for ω_i
t = time
$\omega_1, \omega_2, \omega_3$ = angular velocities about the 1,2, and 3 axes
$\dot{}$ = indicates time diffentiation
(0) = indicates initial conditions

Analysis

In this Note, the stability of a spinning spacecraft during deployment of rigid appendages along one of the transverse axes is investigated using the Sonin-Polya theorem.[1] The system considered[2] is shown in Fig. 1 where the appendages are extended along the '2' axis only. The notations used here follow that of Ref. 2. The independent second-order differential equations [Ref. 2, Eqs. (22) and (23)] for the transverse components of the angular momentum are

$$\ddot{h}_1 - \{\dot{a}_2(t)/a_2(t)\}\dot{h}_1 + a_1(t)a_2(t)h_1 = 0 \tag{1}$$

$$\ddot{h}_2 - \{\dot{a}_1(t)/a_1(t)\}\dot{h}_2 + a_1(t)a_2(t)h_2 = 0 \tag{2}$$

where $a_1(t)$ and $a_2(t)$ are defined as

$$a_1(t) = \{[I_3(t) - I_1(t)]/[I_3(t)I_1(t)]\}h_0$$

$$a_2(t) = \{[I_3(t) - I_2(t)]/[I_3(t)I_2(t)]\}h_0$$

The specific assumption is that h_1 and h_2 are of first-order smallness throughout extension; i.e. $|h_1|, |h_2| << |h_3|$. The extensions are assumed to take place during the interval $0 \leq t \leq t_s$.

The differential equation for ω_1 using the relation $h_1(t) = I_1(t)\omega_1(t)$ is obtained from Eq. (1) as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{I_1^2(t)}{a_2(t)}\dot{\omega}_1\right] + \left\{\frac{I_1(t)}{a_2(t)}\left[\ddot{I}_1 - \frac{\dot{a}_2(t)}{a_2(t)}\dot{I}_1(t) + a_1(t)a_2(t)I_1(t)\right]\right\}\omega_1 = 0 \tag{3}$$

The same procedure is used to obtain the differential equation for ω_2 from Eq. (2). Noting $h_2 = I_2^*\omega_2$, the differential equation for ω_2 is given as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{a_1(t)}\dot{\omega}_2\right\} + a_2(t)\omega_2 = 0 \tag{4}$$

Sonin-Polya Theorem: Let $p(t) > 0$ and $q(t) \neq 0$ be continuously differentiable on an interval $0 \leq t \leq t_s$, and suppose that $p(t)q(t)$ is nonincreasing (nondecreasing) on $0 \leq t \leq t_s$. Then the absolute values of the relative maxima and minima of every nontrivial solution of the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\{p(t)\dot{\omega}_i\} + q(t)\omega_i = 0 \tag{5}$$

are nondecreasing (nonincreasing) as t increases.[1]

As an example, the case of the moving end mass system (Fig. 1a) is considered for the stability analysis. Comparing Eqs. (3) and (5) for ω_1 yields

$$p(t) = \frac{I_1^2(t)}{a_2(t)} = \frac{(I_1^* + pt^2)^2(I_3^* + pt^2)I_2^*}{(I_3^* - I_2^* + Pt^2)h_0} \tag{6}$$

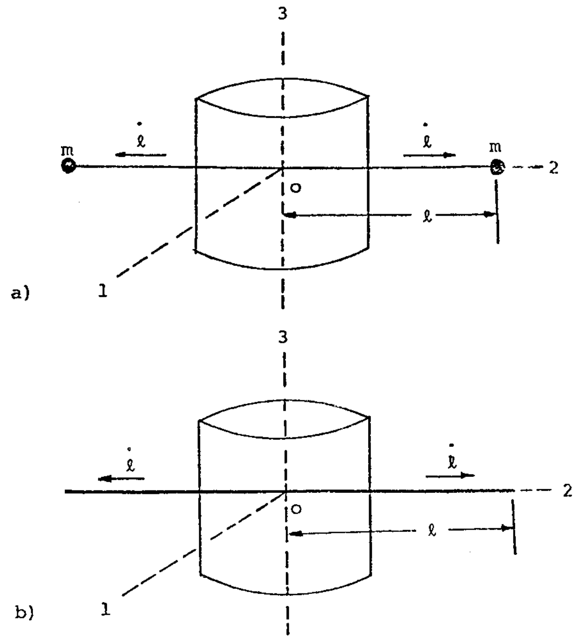

Fig. 1 Two types of telescoping appendages: a) moving end mass system; b) uniformly distributed moving mass system.

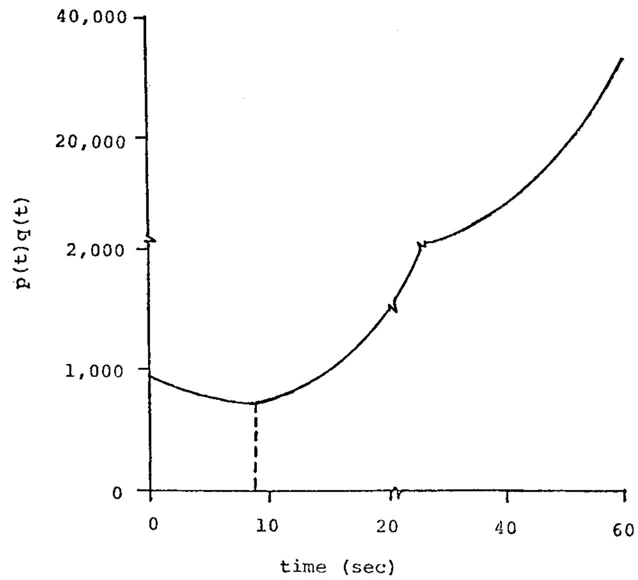

Fig. 2 Variation of $p(t)q(t)$ with time.

and

$$p(t)q(t) = \left\{\frac{(I_1^* + Pt^2)^3(I_3^* + Pt^2)^2(I_2^*)^2}{(I_3^* - I_2^* + Pt^2)h_0^2}\right\} \times \left\{2P - \frac{4I_2^*P^2t^2}{(I_3^* + Pt^2)(I_3^* - I_2^* + Pt^2)} + \frac{(I_3^* - I_1^*)(I_3^* - I_2^* + Pt^2)h_0^2}{(I_3^* + Pt^2)^2 I_2^*}\right\} \tag{7}$$

where $P = 2mc^2$, $c = \dot{l}$, $h_0 = h_3(0)$, and I_i^* $(i = 1,2,3)$ represent the hub moments of inertia.

It is seen that $p(t) > 0$ and $q(t) \neq 0$ for $I_3^* > I_2^*$. The system parameters selected are $I_1^* = I_2^* = 5.0$ slug-ft^2; $I_3^* = 6.0$ slug-ft^2; $m = 0.01$ slug (each end mass); and $c = 1$ ft/s. The assumed initial conditions are $\omega_1(0) = 0.075$ rad/s; $\omega_2(0) = 0.0$; and $\omega_3(0) = 3.34$ rad/s. The variation of $p(t)q(t)$ with time $(0 \leq t \leq 60$ s) is shown in Fig. 2. It is observed that $p(t)q(t)$ is a decreasing function on the interval 0-9 s and is an increasing function after 9 s. Thus, from the application of the Sonin-

Table 1 $\omega_1(t)$ **vs time**

Time, s	$\omega_1(t)$
0.0	0.0750000
1.0	0.0585361
2.0	0.0157344
3.0	−0.0354227
4.0	−0.0714870
4.5	−0.0774900
5.0	−0.0736852
6.0	−0.0387106
7.0	0.0168871
8.0	0.0639985
8.5	0.0758490
9.0	0.0773441
10.0	0.0504268

Polya theorem, it can be inferred that the magnitude of the oscillations of $\omega_1(t)$ is nondecreasing on the interval 0-9 s, and is nonincreasing after 9 s. The response of $\omega_1(t)$ after 9 s will behave as a damped oscillatory system.

The time response of $\omega_1(t)$ on the interval 0-10 s is shown in Table 1. This result is obtained from the numerical integration of the nonlinear equations [Eqs. (1) in Ref. 2] for the case of the moving end mass system. From Table 1, the same result is inferred which was revealed by the application of the theorem. The complete time response of $\omega_1(t)$ during the extension maneuver was presented in Fig. 4 of Ref. 2.

Similarly, the functions $p(t)$ and $p(t)q(t)$ for $\omega_2(t)$ from the comparisons of Eqs. (4) and (5) are

$$p(t)=\frac{(I_3^*+Pt^2)(I_1^*+Pt^2)}{(I_3^*-I_1^*)h_0} \tag{8}$$

$$p(t)q(t)=\frac{(I_3^*-I_2^*+Pt^2)(I_1^*+Pt^2)}{(I_3^*-I_1^*)I_2^*} \tag{9}$$

From Eqs. (8) and (9), it is observed that $p(t)>0$, $q(t)\neq 0$, and $p(t)q(t)$ is nondecreasing. Thus the absolute values of the relative maxima and minima of $\omega_2(t)$ show that it is a nonincreasing function.

Next, the stability of the system in the h_1h_2 plane is considered. Equations (1) and (2) can be rearranged to obtain the following form for h_1 and h_2:

$$\frac{d}{dt}\left\{\frac{1}{a_2(t)}\dot{h}_1(t)\right\}+a_1(t)h_1(t)=0 \tag{10}$$

$$\frac{d}{dt}\left\{\frac{1}{a_1(t)}\dot{h}_2(t)\right\}+a_2(t)h_2(t)=0 \tag{11}$$

Considering Eq. (10) for $h_1(t)$, the function $p(t)q(t)$ is

$$p(t)q(t)=\frac{(I_3^*-I_1^*)I_2^*}{(I_3^*-I_2^*+Pt^2)(I_1^*+Pt^2)} \tag{12}$$

Equation (12) is a nonincreasing function. Hence, the absolute values of the relative maxima and minima of $h_1(t)$ are nondecreasing. The same procedure applied to Eq. (11) indicates that magnitude of the oscillations of $h_2(t)$ is nonincreasing. For the system considered here, the absolute maximum value of h_1 remains in the allowable range $|h_1| \ll |h_3|$ on the interval $0\leq t\leq t_s$.

The above described method can be similarly used to study the stability of the uniformly distributed moving mass system shown in Fig. 1b.

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